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Integrating history of mathematics into the mathematics teaching: a study on concept of variable in elementary algebra in Indian perspective

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Abstract

The present work deals with integration of history of mathematics into mathematics classroom. An elementary topic of algebra, ‘concept of variable’ is taken into consideration. On the basis of cognitive structure of average students of India, a structure is proposed for representation of history of variable into classroom teaching.

Keywords: History of Mathematics, Variable, Classroom Teaching, Elementary Algebra

1. Introduction

History of mathematics is perceived as a human endeavor. Epistemology and history of mathematics is considered suitable for setting mathematical objects. According to Moritz (1993) [17], Andrew Russ Forsyth (1858 -1942) said, “Mathematics is one of the oldest of sciences; it is also one of the most active; for its strength is the vigour of perpetual youth.” [Chapter VII]. In another chapter Moritz (1993) [17] pointed out, J. W. L. Glaisher (1848-1928) said “no subject loses more than mathematics by any attempt to dissociate it from its history” [Chapter VI]. The great French mathematician Henri Poincare (1854 - 1912) even said, “If we wish to forsee the future of mathematics, our proper course is to study the history and present condition of the science.”

According to Rickey (1996) [23] “the careful and judicious use of the history of mathematics”, can become a tool for effective teaching (Fauvel, 1998; Wilson & Chauvot, 2000) [12, 29], and it can help both teachers and students meet the challenges posed by the current reform in mathematics education. Particularly, it can stimulate and develop students’ mathematical communication skills and understanding of mathematical connections, and it can foster students’ appreciation for mathematics (e.g., Arcavi, Bruckheimer, & Ben−Zvi, 1982; Bidwell, 1993; Fauvel, 1991; Reimer & Reimer, 1995; Tzanakis & Arcavi, 2000; van Maanen, 1997; Wilson & Chauvot) [1, 2, 10, 22, 27, 28].

Fasanelli (2000) reported that “There has been interest over several centuries in the relations between the history of mathematics and the teaching and learning of mathematics”. According to Furinghetti (2000a) [14] “the history of mathematics is not a panacea for all the problems of preparing mathematics teachers, … [it] is a good vehicle for reflecting on cognitive and educational problems, for working on students’ conceptions of mathematics and its teaching, and for promoting flexibility and open−mindedness in mathematics”.

On the other hand lack of knowledge about history of mathematics and how it can be integrated into the mathematics classroom is not the only factor accountable for the state of affairs in today’s classrooms across the world. Teachers’ beliefs about the nature of mathematics and about its teaching and learning deeply influence their willingness to integrate history of mathematics into their teaching. If mathematics is seen as a fixed and finished body of knowledge, and if teaching mathematics is seen as the transmission of knowledge from teachers to students, then there is hardly room for the history of mathematics in the teaching and learning process. Silva & Araújo (2001) [24] opined that “if mathematics is seen as one of many forms of knowledge, or even as a kind of cultural manifestation or human activity, then the history of that subject will be meaningful and the study of that history will become a means of better understanding the relationships between mankind and the mathematical knowledge, within a certain cultural context”.

According to Siu (2000) [26] “There are as many different ways to integrate history of mathematics into classroom teaching as there are teachers”. They depend on teachers’ styles, beliefs, and preferences about historical topics. Some authors have suggested models or frameworks for historically inspired classroom teaching. Davitt (2000) [5] strongly supports the use−discover−explore/develop−define model for “constructing accounts of the evolution of numerous mathematical concepts and theories”. This model basically follows the historical sequence of the genesis of mathematical ideas and, according to Davitt, can be used for introducing and developing numerous mathematical concepts, such as functions, negative numbers, geometry, algebra, etc.

Furinghetti (1997) [13] suggested two main streams of integrating history of mathematics into the classroom. She reported “One stream is aimed at promoting mathematics, the other at reflecting on mathematics”. Later, she proposed a model for the main stages in the process of integrating history of mathematics into mathematics teaching (Furinghetti, 2000b) [15]. These stages are named as (1) knowing the sources; (2) singling out topics suitable to class; (3) analyzing the needs of the class; (4) planning the classroom activity, taking into account the availability of means, the aims, and the context of the activity; (5) accomplishing the project; and (6) evaluating the activity. Educationists of the first kind focus their attention on the use of history of mathematics as a tool for designing instruction in such a way that mathematical concepts are introduced in meaningful contexts. Educationists of the second type go further than those of the first type as they seek “what is really pertinent in developing an inspired learning environment. The variety of forms of classroom work that can be used as a means of integrating history of mathematics into mathematics instruction (for example, writings, projects, plays, etc.) urges teachers to use alternative forms of assessment and, thus, to assess their students differently (Carvalho da Silva, 2001; Fauvel & van Maanen, 1997) [1, 11], which is highly valued and recommended by the reform documents (DES, 2001a, 2001b, 2001c; NCTM, 1991, 1995, 2000) [6-8 18-20]. However, the history of mathematic *per se* should not be subject to examination (Fauvel, 1991) [10].

Sierpinska (1996) [25] identified four different ways of viewing the study and use of history of mathematics, which she named *historian*, *epistemologist*, *educationist I*, and *educationist II*. In a nutshell, the historian is mainly concerned about exactly when mathematical developments occurred and why they occurred. Historians perceive the history of mathematics as a subject matter in its own right, and when they use history of mathematics in the classroom, they are overly concerned about ensuring the accuracy of the historical mathematical facts presented. Epistemologists focus their attention on the nature of knowledge in mathematics as well as on how students learn mathematics; therefore, they are concerned with the psychological genesis of mathematical ideas and developments.

Liu (2003) [16] argued ‘Fauvel’s (1991) [10] list of fifteen reasons for including the history of mathematics ….. I propose five reasons for using history of mathematics in school curriculu:

1. History can help increase motivation and helps develop a positive attitude towards learning.
2. Post obstacles in the development of mathematics can help explain what today’s students find difficult.
3. Historical problems can help develop students mathematical thinking.
4. History reveals the humanistic facets of mathematical knowledge.
5. History gives a teacher a guide for teaching.’

Panasuk and Horton (2013) [21] reported on several factors that affect teachers' decision whether include history of mathematics into curriculum and offered some recommendations for teacher professional development and suggestions for further research.

1.1 Objective of the study

It is found that for teaching mathematics, history of mathematics plays a positive role to develop cognitive and affective domains of learners. Therefore, it is important to discuss the historical evidences in textbook and classroom. Objective of this work is to suggest how and which historical backgrounds may be discussed to improve cognitive and affective domains of learners for elementary algebra in Indian perspectives.

1.2 History of Variable

Topic: Concept of variable in elementary algebra

A brief history of mathematics which may be used for this topic is illustrated below:

1. **Rhetorical algebra:** Here equations are written in full sentences. For example, the rhetorical form of x + 1= 2 is "The thing plus one equals two" or possibly "The thing plus 1 equals 2". Rhetorical algebra was first developed by the ancient Babylonians and remained dominant up to the 16th century.
2. **Syncopated algebra**: Here some symbolism is used but which does not contain all of the characteristic of symbolic algebra. For example, there may be a restriction that subtraction may be used only once within one side of an equation, which is not the case with symbolic algebra. Syncopated algebraic expression first appeared in Diophantus' *Arithmetica*, followed by Brahmagupta's *Brahma Sphuta Siddhanta*.
3. **Symbolic algebra:** Here full symbolism is used. It is found that early steps toward symbolic algebra in the work of several Islamic mathematicians such as Ibn al-Banna and al-Qalasadi, though fully symbolic algebra has been developed by François Viète. Later, René Descartes has introduced the modern notation.
4. **François Viète:** (French mathematician) introduced the idea of representing known and unknown numbers by letters, presently called variables, and of computing with them as if they were numbers, in order to obtain, at the end, the result by a simple replacement. François Viète's convention was to use consonants for parameters and vowels for unknowns. After that René Descartes modified the representation of variables as unknowns in equations by x, y, and z, and known by a, b, and c etc.

1.3 Suggested Teaching Plan

1. Development of Algebra will be explored stage wise.
2. Three stages of development (Rhetorical, Syncopated and Symbolic) will be considered.
3. Rhetorical algebra: In this stage algebra was written in full sentences. It was first developed by Babylonians (1800-1700 BC) remain strongly dominant before Diophantus’. In another opinion it was dominant still 1600 AD. For example, at present we write x+5=7, which was written in the rhetorical stage ‘the things plus five equals seven’.
4. Syncopated Algebra: Diophantus was a Greek mathematician. He is known for having written *Arithmetica*, a treatise that was originally thirteen books but of which only the first six have survived. In *Arithmetica*, he is the first to use symbols for unknown numbers as well as abbreviations for powers of numbers, relationships, and operations; thus he used what is now known as *syncopated* algebra. The main difference between Diophantine syncopated algebra and modern algebraic notation is that the former lacked special symbols for operations, relations, and exponentials. For example x^3+2x^2+5, in syncopated algebra ‘unknown cube, unknown square 2, 5’. Although, Syncopated algebraic expression first appeared in Diophantus' *Arithmetica*, it was followed by Brahmagupta's *Brahma Sphuta Siddhanta*.
5. Symbolic Algebra: Here full symbolism is used. Presently we are using this type of algebra. Approximately after 1600 CE, this stage of algebra was started.
6. Concept of Variable: Babylonians solved problems in variables without symbols. They used only words to denote such numbers, and that is why algebra has been referred to as rhetorical algebra. Around 1600 B.C., in algebraic problem, unknown is found in Egyptian papyrus.

7. Indian mathematicians: Brahmagupta first used variable, after that Aryabhatta I, Brahmagupta and Bhaskaracharya II were some the eminent ancient Indian mathematicians who propounded and perfected the methods of ‘beejganita’, ‘the mathematics of letters’.
8. Variable at present: François Viète (French mathematician) introduced the idea of representing known and unknown numbers by letters, presently called variables, and of computing with them as if they were numbers, in order to obtain, at the end, the result by a simple replacement. François Viète's convention was to use consonants for parameters and vowels for unknowns. After that René Descartes modified the representation of variables as unknowns in equations by x, y, and z, and known by a, b, and c etc.

1.4 Concluding Remarks
This approach of classroom discussion may help students to understand how the concept of variable originated from past. Teacher may use instructional aids (maps, photos of mathematicians, ppt etc.). Topic wise discussion of history of mathematics plays an important role in learning mathematics because it creates interest, positive attitude and respect to the subject which are finally strengthen the cognitive and affective domains of the learners.

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These 11 essential strategies in teaching mathematics can help.Â These 11 essential strategies in teaching mathematics can make this your classâ€™s best math year ever. 1. Raise the bar for all. It can be a challenge to overcome the socially acceptable thought I was never good at math, says Sarah Bax, a math teacher at Hardy Middle School in Washington, D.C. Rather than being born with or without math talent, kids need to hear from teachers that anyone who works hard can succeed.Â Testing is not something separate from your instruction. It should be integrated into your planning. Instead of a quick exit question or card, give a five-minute quiz to confirm students have mastered the math skill covered in the dayâ€™s lesson. Mathematical knowledge includes knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a disciplineâ€”in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof. In our use of the term, knowledge of mathematics includes consideration of the goals of mathematics instruction and provides a basis for discriminating and prioritizing those goals. Knowing mathematics ...Â For example, a study of prospective secondary mathematics teachers at three major Elementary mathematics consists of mathematics topics frequently taught at the primary or secondary school levels. There are five basic strands in Elementary Mathematics: Number Sense and Numeration, Measurement, Geometry & Spatial Sense, Patterning & Algebra, and Data Management & Probability. These five strands are the focus of Mathematics education from grade one till grade 8.