

# An analysis of mathematics textbooks and reference books in use in primary and secondary schools in England and Wales in the 1960s

*John Breakell*

## Abstract

Teaching mathematics in the United Kingdom in the late 1950s and early 1960s, whether at primary or secondary level, invariably involved the use of a textbook and an exercise book. At secondary and primary school level, series of mathematics texts were popular with teachers because they provided a ready-made curriculum extending over two or more years. It is suggested that data about text and reference books available for use in schools during this period reflects with some degree of accuracy, both the content and the methods of teaching favoured by the teachers of the day. It is also suggested that the popularity of certain textbooks is mirrored by the number of reprinted editions.

## Background

The period of this review, defined as the early and middle 1960s, is the time when the government-inspired Schools Council for Curriculum and Examinations (which potentially had the strength to generate substantial change in the curriculum field, including mathematics) was formed, and when a major Primary Mathematics Project, supported by the Nuffield Foundation, and the School Mathematics Project (SMP), targeted on secondary stage education, and supported by industry, were established. Other initiatives were coming on stream towards 1965, for example the Midlands Mathematics Experiment (MME) and somewhat later, the Mathematics for the Majority Project (MMP), supported by the Schools Council for Curriculum and Examinations.

The materials produced by these projects began to have some impact on the teaching and learning of mathematics in the latter half of the 1960s. But even before that there was evidence that a groundswell of change was emanating from professional organisations such as the British Mathematical Association, which commissioned reports in the 1950s and 1960s which seriously questioned the content of the mathematics curriculum of the day and made recommendations for improvement in the future.<sup>1</sup>

In the early 1960s there was much talk of the need to introduce 'modern mathematics' into the curriculum. It was held that this impetus reflected concern in America about the strength of Soviet

progress in the space race and that the content of 'modern mathematics' was felt to represent a much more powerful tool for use in the scientific field than traditional mathematics. This suggestion has been challenged. A.B. Evenson considers that there were many reasons for the (then) current revolution in mathematics, not least the apparent inadequacies of traditional mathematics and the fact that wider applications of mathematics had increased considerably in the recent years; a need arose for new and more powerful instruments, resulting in the creation and use of new kinds of mathematics.<sup>2</sup> In Britain a new approach to the way in which mathematics was taught and presented emerged during this period, with a stress on pupils being actively involved in their learning, especially at primary school level. The effect of all these factors began to be seen in the content and presentation of textbooks published in the 1960s.

## Methodology of the survey

A sample of 176 school mathematics books from the University of London Institute of Education Library available in the 1960s was examined. The Library was used as the source for this enquiry since it has held a comprehensive collection of text and reference books focusing on a large range of school subjects for many decades. Whilst the sample was not exhaustive, it was likely to be representative of the mathematics books available in primary and secondary schools at this time. The investigation endeavoured to determine which of these mathematics books, whether text or reference, and whether for use in primary or secondary schools, were representative of a modern or a traditional approach to content and whether they were modern or traditional in presentation. Of the 176 books examined, 15 were individual texts, while the remaining were defined as being members of a series, of which there were 36 – a series being defined as having two or more parts or stages. In summary, there was thus an overall sample of 51 items. A further survey was conducted to ascertain how many reprinted editions of books or series of books were published, since this gave some indication of popularity. This information is incorporated in the body of the paper.

Certain characteristics of the books were identified with some confidence. Clearly, not least because of the

<sup>1</sup> For example: Mathematical Association, *The teaching of mathematics in primary schools* (London: Bell, 1955); Mathematical Association, *The teaching of mathematics in secondary modern schools* (London: Bell, 1959).

<sup>2</sup> A. B. Evenson, *Modern mathematics: introductory concepts and their implications* (London: Sir Isaac Pitman & Sons, 1962), pp. 7-8.

information in the title or in the foreword, a text was intended for use in either primary or secondary schools, although the divide was bridged in terms of content and of intellectual challenge to pupils in a very few of the texts. The definition of other characteristics attributable to a series or to individual books was attempted in the following manner. Firstly, any mathematical text was defined as being 'traditional' where the content consisted of the sequential examination of a number of well-established topics. At the primary stage, examples were the four operations of arithmetic - addition, subtraction, multiplication and division - applied in a range of measures; at the secondary stage, a number of topics in arithmetic, or trigonometry, or algebra or geometry, such as determining factors in an algebraic expression, proving a theorem in geometry, determining the length of a side of a figure using trigonometrical tables and simple and compound interest in arithmetic.

Secondly, some texts published in the early to mid 1960s included a range of new topics - for example - work in a variety of number bases, but especially related to the binary system and its application in computing, data collection and representation, probability, the concept of a mathematical set and rotational geometry. These items were representative of the 'new' or 'modern' mathematics. This element of the content was defined as modern, but since some traditional topics were still represented in such texts, the overall content was defined as 'enhanced'.

Thirdly, texts were defined as being traditional in presentation where, consistently, there was, first of all, a general exploration of an individual topic, followed by some worked examples and a large number of practice exercises. Fourthly, an increasing number of books were representative of a style of approach which, today, could be defined as 'user friendly'. This was facilitated by the use of colour in the printing of the words or illustrations, by the placement of comments and stimulus questions as the topic was developed. The methodology of approach in this type of book was defined as 'enhanced'.

It is not possible to define any of the books in terms of just one of the four characteristics, since the content has to be presented to the reader in one form or another, be this in a traditional or in an enhanced mode. The books have first been divided into having either a primary or a secondary orientation and then subdivided into four groups, as depicted in Table 1 below. Table 2 quantifies the frequency of reprinting of texts or series.

**TABLE 1**  
**Individual texts and series: categories**

Secondary school texts and series		
	<i>traditional content</i>	<i>enhanced content</i>
<i>traditional approach</i>	7 series 8 texts	2 series 0 texts
<i>enhanced approach</i>	0 series 0 texts	3 series 2 texts
	TOTAL 12 series and 10 texts	
Primary school texts and series		
	<i>traditional content</i>	<i>enhanced content</i>
<i>traditional approach</i>	11 series 4 texts	0 series 0 texts
<i>enhanced approach</i>	11 series 1 text	2 series 0 texts
	TOTAL 24 series and 5 texts	
	GRAND TOTAL 36 series, 15 texts	

**TABLE 2**  
**Individual texts and series: reprints**

Secondary school texts and series		
	<i>traditional content</i>	<i>enhanced content</i>
<i>traditional approach</i>	7 series, 1 reprint 8 texts, 4 reprints	2 series, 1 reprint 0 texts
<i>enhanced approach</i>	0 series 0 texts	3 series, 1 reprint 2 texts, 1 reprint
	TOTAL 3 reprints of series and 5 of individual texts	
Primary school texts and series		
	<i>traditional content</i>	<i>enhanced content</i>
<i>traditional approach</i>	11 series, 3 reprint 4 texts, 1 reprint	0 series 0 texts
<i>enhanced approach</i>	11 series, 1 reprint 1 text	2 series 0 texts
	TOTAL 4 reprints of series and 1 of an individual text	
	GRAND TOTAL 7 reprints of series, 6 of individual texts	

### Detailed analysis of the books

#### *Texts with traditional content and traditional approach - secondary stage*

Perhaps the most distinguished example in this category was Loney and Grenville's book, entitled simply *Arithmetic*, first published in 1906, which covered a whole range of topics in arithmetic - vulgar fractions, percentages, profit and loss, square roots, for example. The text was extremely dense, containing what can only be described as a solid mass of exercises and problems for students to address. The book was recommended by the Mathematical Association in the early 1900s to prepare pupils for the Oxford and Cambridge Junior Local examinations, but its very popularity, at least up till the early 1960s, suggested that it was also very useful in preparing for the national General Certificate of Education Ordinary Level examination (GCE 'O' level), set for pupils in England and Wales at about the age of 16 years. Another text with an extremely long life was Hall and Stevens' *A School Arithmetic*. It was first published in 1908 and was still used in schools in the 1960s.

A title which reflected current concern with gender differentiation was *Practical Arithmetic for Girls* written by R.E. Harris. It ran to four reprints between 1953 and 1962 and was directed towards the apparent needs of secondary school girls who were coming up to the school leaving age (at that time, 15 years). The content reflected an expectation that women would have to become familiar with arithmetical requirements associated with cooking, with earning a wage, with computing electricity and gas bills, with planning a budget and with the functions of the Post Office.

As would be expected, a large number of texts or series of texts were produced to prepare students for examinations at 15 or 16+. In this category most make no pretence of doing other than addressing the traditional content in a traditional manner. *Ordinary Level Algebra* by T.H. Ward Hill, published in three parts in 1960, with Part One reprinted in 1965, prepared students for the GCE 'O' level Algebra paper by considering a standard range of topics such as factors, quadratic equations, graphs and their interpretation, and gradients. Similarly *O Level Arithmetic* by A. Keith, published in 1963, concentrated on the whole of the requirements of the arithmetic paper at GCE 'O' level. There were numerous exercises related to a range of topics such as fractions, decimals, logarithms, square roots, the metric system and numerical trigonometry.

Some authors chose to write a comprehensive course in mathematics, in contrast to those who offered coverage of a single element of the subject. For example, R. Walker produced a series in five parts entitled *School Mathematics* between 1960 and 1963 to help prepare students for the four mathematics papers at GCE 'O' level. Topics in arithmetic, algebra, geometry, statistics and trigonometry were considered consecutively in each book.

Other texts published at this time were essentially collections of tests and were designed to meet the revision needs of pupils just prior to sitting the GCE 'O' level examination. *General Certificate Tests*, by W.A. Gibby, offered a number of 40-minute tests, the format of which reflected that of the examination itself -- questions and problems which ranged from the short and simple to the longer and more complex.

A number of texts and series of texts in the traditional content/traditional approach category did attempt some leavening in the manner in which the material was offered to the reader. *Cornerstone Mathematic* by R.E.Harris, published in three parts between 1961 and 1963, addressed work in arithmetic, algebra and geometry. The books certainly utilised exercises and problems in the traditional mode, but attempted to be rather more 'user friendly' by the interspersing of interesting and amusing pen sketches, illustrating various features of mathematical procedures.

*Modern Arithmetic*, written by C.H. Hopkins and published between 1962 and 1963, consisted of an introduction and four stages; Stage 4 prepared students for the academically less taxing Royal Society of Arts examinations, taken by students at about 16 years of age. The text began by looking at the history of numbering and suggested the utilisation of the 10 x 10 '100 square' to facilitate exercises in mechanical arithmetic. The author made an attempt to relate the material to everyday life, by including some practical measuring activities and by, for example, acknowledging the way in which darts players subtract when computing scores. Nevertheless, the major component in the texts was the traditional diet of exercises and problems.

Shaw and Wright produced *Discovering Mathematics* in four books between 1960 and 1963. Despite the title, the series consisted of a large number of routine practice exercises related to themes in arithmetic, algebra and geometry followed by a series of tests; there was a stress on enabling pupils to grasp the 'quick technique' to achieve solutions to problems. What made this series slightly different was that there is some evidence of a wish to communicate more effectively with the reader through the use of informal language and amusing illustrative diagrams.

#### *Texts with traditional content and traditional approach - primary stage*

During the period under review, the content of most primary stage mathematics books remained essentially traditional; however there was discernible evidence from the mid 1950s that the approach to the presentation of material was beginning to change. Colour was used in some of the printing; explanations about the significance of topics were offered and 'pen and ink' sketches introduced.

Among the 126 primary stage books reviewed in this enquiry a few fell quite clearly into the traditional content/traditional category approach. One series in four books entitled *Two Grade Arithmetic* by Lovell and Smith, published in 1956, was extremely popular. Aimed at the Junior age range (7 - 11 years), it had a comprehensive coverage. A large number of examples were given and practice was achieved through a vast number of exercises, with a reserve of more exercises in the accompanying teachers' books. Larcombe produced *A New Arithmetic for Primary Schools*

between 1953 and 1962 in four books. The material consisted of a range of exercises and problems related to a large number of topics in Junior stage arithmetic.

A curious example of work in arithmetic and English produced under one cover is *A Junior School Revision Course, Arithmetic and English* by W.S. James, published in 1961. The material, insofar as the arithmetic was concerned, related to the application of rules in practice exercises and problems. The author, however, missed the opportunity of linking together any of the material in English and arithmetic in the two sections of the book.

There was a sizable market for books providing revision opportunities and tests in preparation for the 11+ examination set at the end of their primary stage education by local education authorities in England and Wales for several decades - the results of which determined the type of education a pupil would be offered at secondary school level. A typical example was to be found in K. Anderson's books *Arithmetic Tests for 3rd year Juniors* (there was a companion volume for 4th year pupils), published in 1961, which offered revision and practice through tests of mental arithmetic and the application of mechanical skills in exercises. A series of four short books which had a similar purpose was produced by V.A. Carter in 1961 for Junior school pupils under the title *I Work Out*; the material consisted of a collection of activities involving mental arithmetic, practice exercises and problems.

Turning now to the range of books where the content was fundamentally traditional, but leavened through the use of colour and a more interactive style, a number of texts and series of texts come immediately to mind. Dora E. Whittaker was responsible for writing a series of four books for Junior school pupils entitled *Mathematics through Discovery*, published in 1965. A Teachers' Book was also published in the same year. The cover utilised two additional colours, one within the text. The series was very popular and ran to three reprints of the basic four books by 1967 and two of the Teachers' Book. The latter was regarded, in part, as a guide and contained suggestions for the treatment of topics with pupils. Arithmetical division was introduced through continuous subtraction, rather than as a quick technique to be mastered. Teachers were advised to discuss mathematical concepts with children and then to encourage them to consider the applications of appropriate mathematical techniques in real life situations. Book 3 referred to the work of Galileo and opportunities were given for practical activities in this context in class; number bases other than ten were introduced. Nevertheless the overall content was traditional and there were many exercises to be undertaken by pupils. What was clear, however, was that the author had made considerable efforts to present mathematics in a way that pupils would find the subject at least tolerable, and possibly exciting and rewarding.

Flavell and Wakelam's *Primary Mathematics - An Introduction to the Language of Number* was arguably the most popular series of mathematics books used in primary schools in England and Wales in the 1960s and early 1970s. Books 1, 2 and 3 were produced in 1960 and 1961, utilising one additional colour in the text; each book was matched by a Teachers' Book and an answer book. A short supplement of 16 pages was produced in 1969 to anticipate the change to decimal currency in the United Kingdom in 1971. As with Dora Whittaker's Teachers' Book, Flavell and Wakelam's devoted much of the text to a discussion of the relevance and importance of a particular mathematical topic and gave guidance on its introduction to children.

The books were easy to handle and presented an almost square appearance, at 21 cm by 17 cm for the principal texts, - quite different from the 'portrait' style presentation of the traditional textbook. One additional colour was used on white in the printing. A range of topics of a mainly traditional kind was introduced and there was a limited amount of 'modern' content; there were exercises, some problems, but also some inferential work for pupils to undertake. Techniques such as multiplication and division were explained at some length with practical examples, employing, for example in relation to multiplication, a picture of a milk crate, with its rectangular lattice of 8 x 6 spaces.

The authors went on to publish a number of supplementary books. Typical titles were *Way In* (1962), an elementary book on number, reprinted twice, and *Lines and Shapes* (1963). Reflecting the newly emerging interest in modern mathematics, Flavell and Wakelam produced *Introduction to Sets* in 1965.

For the Junior age range in the *Teach Yourself* series, *Arithmetic Itself* was published by Burn and Tamblin in 1962 and utilised two additional colours. Although the content was traditional and addressed a large number of topics in arithmetic, the approach was modified. Whilst offering the usual diet of worked examples, the authors made a major effort to speak directly to the reader by way of encouraging a degree of interaction. There was much use made of the pronouns 'I' and 'you' in the text. The suggestion was made that pupils could work by themselves on the material, but on balance this would seem to be difficult and explanations would still be required by the teacher.

T.H. Flanagan produced a set of 12 short (16 pages) books for use in junior schools in 1964 and 1965 entitled *Topics in number*. Each booklet (which utilised one additional colour), addressed a particular issue, such as 'Weights and Weighing, Measuring Liquids and Spending Money'. The content was traditional, with simple and then more complex problems, but the approach, through the use of short interesting explanatory statements, followed by the problems, represented a slight modification to the traditional approach.

A final example where the content and approach were fundamentally traditional yet with some attempt to modify the latter is seen in R. Harris' book entitled *Angles* published for use in the top of the junior age range and in the lower age range of the secondary school in 1964. The book was described as a programmed text and advocated practical activities involving ruler and compass in measuring angles, with answers to be entered in small rectangular boxes in the book.

#### *Texts with enhanced content and traditional approach - secondary stage*

Few books fell into this category at secondary school level, but those that did tended to be very popular at the time. Reflecting the concerns of the early 1960s about the inadequacies of mathematics teaching and their implications, some authors set out to provide fresh content, identified as 'modern mathematics'; the material was however presented in a traditional manner, often with dense text and with many practice exercises.

A group of teachers working in the Midlands, led by Cyril Hope, had produced an experimental GCE 'O' level course in mathematics. The title was simple enough - *The Midlands mathematics experiment* - and Books 1, 2 and 3 were published in basic form between 1963 and 1965 for trialling. A summary of the contents of the three books gives some indication of the importance placed on the inclusion of new material. Book 1, for example, addressed different number bases and the connection through the binary system to computer usage; then navigation, fractions, logarithms, decimals, sets and set language, negative numbers, areas and points of the compass; Book 2 looked at estimation, transformations, probability, systems of units and further work in sets. In Book 3, students were introduced to modular arithmetic, matrices, Boolean algebra and its use in electrical circuitry, integration and differentiation in calculus and vectors. At a later stage a definitive Volume 1 was produced to serve the requirements of both the GCE 'O' level and the new Certificate of Secondary Education (CSE) examinations, the latter widely used and flexible in operation, mainly taken by pupils of average ability in schools in England and Wales. Volume 2 was published in two editions, the first to provide for the needs of years three and four of the GCE 'O' level mathematics course, to be followed by Volume 3 to complete the 'O' level course and to bridge the gap to sixth-form work; the second, another Volume 2, met the further needs of CSE pupils. This was a clear attempt to modernise the content of mathematics teaching, but the presentation of the new diet was traditional, despite some excursions into activities such as paper folding.

One of the most popular series of the 1960s was that produced by Mansfield and Thompson entitled *Mathematics - a new approach*; it was published in five books between 1962 and 1966. While a large

proportion of the content was certainly modern, the approach remained traditional. Book 1 was used as the core textbook for grammar schools during the early stages of the implementation of the Schools Mathematics Project (SMP). A review of the topics covered in each of the five books underlines the fact that not only was new content offered but often the descriptive terminology of some of the traditional material was modified, for example, words such as 'coordinates', 'tabulation', 'median', 'percentile', which were to become commonplace in later years, were used comprehensively for the first time in these books. Topics in Book 1 included binary arithmetic, fractions, lattices, statistics and collation, primes, commutative and non-commutative algebra, rotation, triangles, tessellations and symmetry, whilst in Book 2 linear programming, pie charts, histograms, probability, the normal distribution, maps and surveying, triangulation and simple trigonometry were introduced. Book 3 addressed the notion of sets with reference to intersection, union of sets and Venn diagrams, transformations, topology, analogue computing, mathematical grouping, rings, fields and matrices.

Topics in Books 4 and 5 included mathematical mapping, further Boolean algebra, logic, flow diagrams, computer programming, isometrics, transformations and rotations, vectors, best straight line and standard deviation, all of which were relatively unknown in typical school curricula of the day. However, all the books retained a proportion of traditional content, such as axiomatic geometry, quadratic equations, decimals and the binomial and remainder theorems.

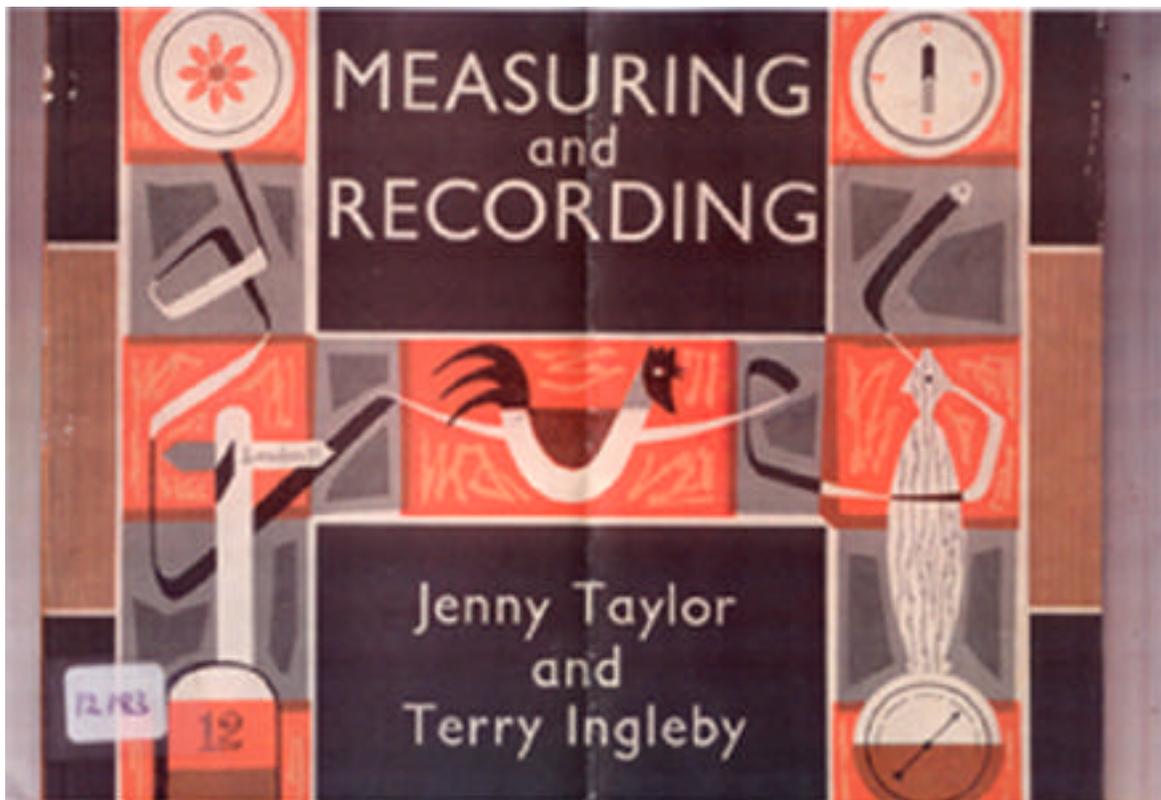
The principal aim of the SMP was to develop a modern course applicable to everyday life. A key element in its success, from the point of view of teachers, lay in its completeness. Between 1963 and 1972 it provided texts, Books 1 to 5 and Books A to H respectively, which furnished a ready made two three or five years' course for pupils, together with teachers' handbooks for each text. The former series was designed essentially for intellectually able pupils in selective schools or top streams in the newly developing comprehensive schools in England and Wales aiming to take the GCE 'O' level examination, whilst the latter series, modified from the former, was for pupils who were preparing for their CSEs. SMP achieved immense popularity and undoubtedly changed the face of mathematics teaching at secondary level. The content consisted of a mix of the modern and the traditional, while the presentation, though principally traditional, was leavened by the use of lively text, stimulus questions and opportunities for interaction.

*Texts with enhanced content and traditional approach - primary stage*

No books in the sample were found in this category at the primary stage.

*Texts with traditional content and enhanced approach - secondary stage*

No books in the sample were found in this category at the secondary stage.



Front cover from *Measuring and recording* by T. Taylor and T. Ingleby, published in 1963 by Harlow

*Texts with traditional content and enhanced approach - primary stage*

*Measuring and Recording* by T. Taylor and T. Ingleby, published in 1963, represented a new departure in the way in which information was given to pupils, in this case at the Infant stage. The book was large, attractive and colourful and was presented in 'landscape' format. It used one additional colour on the cover and two within the text. No exercises, worked examples or problems were shown. The book illustrated a relationship between the spoken or written word or symbol and its representation; thus '5' and a drawing of five tomatoes. Attention was concentrated on illustrating facets of a range of numbers from 1 to 10 and from 10 to 100. The difference between cardinal and ordinal numbers was noted, using a staircase and a running race to illustrate the latter. Within the 24 pages, the authors managed to present, in a most attractive form, illustrative sketches depicting weighing and the use of scales, measuring with tape and ruler, signposts and mileposts, maps and compass points, clocks and calendars and temperature measures. The book was aesthetically pleasing whilst conveying, in simple fashion, simple ideas about the concept of number and about measurement.

L.G.W. Sealey produced a number of books from 1961, under the general heading *Some important*

*mathematical ideas*, aimed principally at the middle to top Junior age range. The covers of these books exhibited symmetrical geometric designs in different colours and shading, the general effect of which was pleasing to the eye. By way of example Book 'C1', published in 1961 and reprinted in 1962 and 1965, with a revised version in 1967, reviewed, in 50 pages, a number of traditional topics, such as series in number, shapes, fractions, regular and irregular areas, scale and plans, heights and shadows, gear wheels and their working, bouncing balls, curve stitching, line charts, pairs of lines, square numbers, averages, percentages and ratio. The approach to the work was innovative. Children were given a series of practical tasks and expected to work in pairs to discuss ideas and possible solutions. There were no formal exercises or problems in the traditional sense; there were many questions for children to ponder and the whole could be seen as an exciting challenge. Other titles in this series published in the early 1960s were *Using mathematics*, *Mathematics around us*, *Learning about ourselves*, *More mathematical ideas*.

Another popular series at this time was *Let's explore mathematics* in four books by L.G. Marsh, published for use in junior schools between 1965 and 1967. Topics in mathematics, principally of the traditional kind but with a few which were modern, were dealt with in a random fashion. Book 3, for example, dealt

with estimation, weighing and measuring, plans and elevations, mathematical grouping, fractions, Archimedes and volumes, line graphs, the abacus, Napier's Bones, magic squares, the sieve of Eratosthenes, building bridges and solid figures using nets. In Book 4, patterns in number and different number bases (including reference to the binary system relating to computing) were discussed, together with coordinates and maps, the measurement of turning (rotational geometry), sets and relationships, including intersection and union, and the history of number. The books were well illustrated with bold colours and black and white drawings. Pupils were expected to work with a partner in order to discuss the tasks, before undertaking the related practical work.

Razzell and Watts produced a set of books in 1964 under the general title of *Mathematical topics*, each of 32 pages. All were attractively presented, using two additional colours on the cover and in the text. The six books with individual titles such as *Circles and curves*, *Symmetry*, *Probability* and *A question of accuracy* were aimed at the Junior age range. Pupils were introduced to the idea of probability through a story about the predicted appearance of Halley's comet; they were invited to toss coins to achieve some understanding of the probability of an event occurring. Subsequently, through practical activities, pupils gained an understanding of the notion of a sample and of a random sample by taking beads from a box, before considering the application of sampling in opinion polls. Issues such as 'margin of error', 'estimating large numbers' and 'long range weather forecasting' all found a place in this book.

These books did not seriously pretend to be complete textbooks, rather topic or theme books which would be seen as complementary to an existing course scheme. A minority of the latter were beginning to show some signs of a change in approach whilst still maintaining a traditional content with the inevitable large number of exercises for pupils to undertake. Under this heading, one of the most popular series of the day was entitled *Oxford Junior Mathematics*, written by E.M. Williams and E.J. James. The 5 books of the series were aimed at the Junior school age range and first published in 1962; a set of teacher's books were also available. The text was printed in black, the covers in one additional colour. The evidence of change manifested itself in the way in which the authors tried to catch the imagination of the readers by inserting a short introduction to each book. In Book 5, for example, in alluding to 'Mathematics Around Us', the authors described the pattern of a honeycomb and a snow crystal and later refer to the elliptical orbit of the earth and other planets around the sun.

*Beta Junior Arithmetic*, by Goddard and Grattidge and published in four books between 1962 and 1963, also offered a traditional content, addressing topics in number and measurement, money, time and shape, but the presentation was characterised by the use of additional colour in the text and two extra colours for

the covers. Four teachers' books were linked to the basic pupil's books; these contained a great deal of information for the teacher with explanations of the principles involved in the topics and guidance as to how they might be introduced and developed.

J.G. Saunders wrote *Mathematics alive* in three books in 1964, each with a linked teachers' book, the target audience being from second to fourth year junior school pupils. One additional colour was used on the cover and in the text. None of these books exceeded 56 pages, and as with many of this type, developed a series of topics. In Book 1, under the heading of number, reference was made to the abacus, to notation and to patterns in tables. Geometrical shapes and symmetry were investigated; train and car journeys linked to time were addressed under the general heading of 'travel'. Elementary graphs were introduced, as was work involving the calendar. In Books 2 and 3 the same titles were utilised but the work became increasingly challenging, for example, in addressing some simple mechanics involving gear wheels and pulleys. The teacher's books gave extensive explanations of the subject matter and suggested useful questions which could be put to children.

*Mathematics alive* broke new ground by injecting the traditional content with a considerable amount of practical and meaningful activity for children and a limited number of exercises and problems.

Two series principally for use with pupils in infant schools, aged from 5 to 7 years, pursued their objectives in different ways. Firstly, *Discovery mathematics for top infants and lower juniors*, written by 'Willbrook' in the early 1960s and secondly *Four way number* by W.M. Ferrier, published in 1962. The former, using one additional colour on the cover and two in the text, was an umbrella title for a number of separate workbooks for children illustrating length, weight, time and capacity. In the section devoted to weight children were invited to collect common but different items and to experiment with quantities of one against quantities of another using a simple balance, out of which experiment the concept of balancing would emerge. Non-standard weight measures were used initially, eventually moving on to standard measures. However, a high degree of reading fluency would be required for the workbooks to be used efficiently by children.

The activities in *Four way number* were based on a series of eight simple stories to be read by children; additional equipment comprised a flannel graph and some toys. A teachers' book was produced at the same time. A series of linked workbooks for pupils required them to recognise and practise using the names of common numbers and then to answer simple questions which would lead them to an understanding of addition and subtraction. The content was undoubtedly traditional, but the approach was novel and would appeal to children, even if the operation was somewhat time consuming for the teacher.

Books under the general heading of *Colour factor mathematics* by H.J. Thompson, published between

1962 and 1964, provide an interesting conclusion to this section. In the early 1960s there was considerable interest in utilising coloured rods of different lengths and colour directly proportional to the numbers 1 to 12, or the multiples of these numbers, as a teaching aid to facilitate pupils' understanding of number relationships. The rods were fully integrated into a system, so that for example, a 3 rod plus two other 3 rods would equate in length to a 9 rod. The manufacturers of Colour Factor material believed that since (at that time) the United Kingdom utilised a duo-decimal system in money and length they were justified in using this range, whereas another manufacturer's 'Cuisenaire' material used the decimal system, with rods of proportional length 1 to 10. The use of the Colour Factor materials was linked to a set of five texts to be used in the first five years of the primary school. Essentially the content of the books was traditional, but the approach, using the rods in all kinds of computation was very new and for ten to fifteen years caught the imagination of many headteachers and teachers as a means of helping pupils understand number. Each pupils' book contained a detailed series of practical activities. The first year material was written in two books of 62 pages each entitled *Prenumber Mathematics* and *Number and Basic operations*. A teachers' book, giving the answers to the exercises, was published in 1963.

*Texts with enhanced content and enhanced approach - secondary stage*

Possibly the most lively series of books which addressed topics in modern mathematics was that entitled *Contemporary school mathematics (CSM)*, produced by staff at the independent selective St. Dunstan's College in south-east London. The general editor, Dr Geoffrey Matthews, became in 1964 director of the Nuffield Primary Mathematics Project. The principal characteristic of the books is the sheer enthusiasm and excitement which the authors gave to their work. The eight booklet titles are as follows:-

- Matrices 1 and 2
- Sets and Logic 1 and 2
- Computers 1 and 2
- Shape size and place
- Introduction to probability and statistics.

All were published in 1964 and varied between 68 and 96 pages in length. The covers were colourful, utilising a simple geometric design made up of coloured dots. The text explained each element of the topic in straightforward language and often in amusing terms. In considering the rationale of a flow chart, for example, the point was made that socks, under normal circumstances, must be put on before shoes, not the other way round. Despite this light-hearted approach a serious attempt was made to explain the significance of the new material to the reader. There were worked examples and a number of short exercises. The pace of

the introduction of the material was quick and the material itself was certainly demanding intellectually. The booklets offered a basic introduction to ideas and concepts in modern mathematics and would be adequate to cover these elements in preparation for the GCE 'O' level examination and to a limited extent for the GCE 'A' level examination, taken by pupils at 18 years of age intending to proceed to higher education.

Doris Bass produced a series of substantial books between 1963 and 1966 under the title *Mathematics*, each varying in length from 228 and 278 pages. The first three prepared for general work in mathematics, with Books 4 and 5 especially directed towards work for the GCE 'O' level examination. The texts set out to develop a contemporary study of mathematics with particular attention being given to some of the newer topics; exploration by pupils as part of task activities was encouraged. Book 1, for example, addressed the need for place value and the use of the zero symbol and assessed potential answers in terms of odds and evens following computation. Book 2 introduced sets and Venn diagrams, ordered pairs, the Moebius Strip, bearings and the compass, whilst Book 4 offered a study of different number bases, of powers and of matrices. There were however many traditional exercises for children to work on, but the demonstration of ideas and concepts which both precede and follow the exercises and problems were presented to the reader in an attractive manner. The books constituted a fulsome presentation of course material featuring both new and traditional mathematics; they were popular during the 1960s and early 1970s.

The Scottish tradition of teaching and learning had always been respected and this was shown by the popularity of the publication in 1956 by the Scottish Mathematics Group of the first of a series of seven books under the general title *Modern mathematics for schools*, preparing pupils for the Scottish Certificate of Education, similar in status to the General Certificate of Education 'O' level examination used in England and Wales. Many teachers throughout the United Kingdom found the series useful in preparing for the GCE 'O' level examinations, and Book 1 was reprinted within the year of its first publication. A selection of some of the content of this book illustrates the attention given to modern mathematics topics, for example, sets, equations, commutative and associative distribution laws and inequalities. The content overall is best described as a mixture of traditional and modern.

E.H. Lockwood's *A book of curves* published in 1961 was extremely popular and reprinted twice in the 1960s. As the title suggests the book studied a number of curves - the parabola, ellipse and hyperbola. The text was fairly dense and was accompanied by a large number of elaborate diagrams; practical activities for the reader to undertake were included.

The final entry in this section is Thyra Smith's *The story of measurement* published in 1955. Although not a text in the conventional sense, it did address a wide

range of issues associated with measurement in a revealing and interesting manner, particularly noting the reasons why man measures and the degree of accuracy required in a specific context. The author made the point that this was related to the specific needs of the measuring event, for instance as applied in measuring a spark plug gap, in working with scale models, and in investigating the divergence of magnetic north from true north. The whole was presented rather as a story in anticipation of holding the reader's attention.

#### *Texts with enhanced content and enhanced approach - primary stage.*

Possibly the most formidable example at the primary level of a number of books where both the content and the approach could be regarded as enhanced, was that produced by the Nuffield Primary Mathematics Project from the mid 1960s. None of these products could strictly be regarded as texts, all were in fact teachers' guides. The first, *I do and I understand* endeavoured to convince teachers that hands on experience for children was more valuable than repetitive formal teaching. *Pictorial representation* showed teachers how data children had collected could be organised and recorded in various cogent forms, and described teaching points which can arise from this form of activity. In all the books ideas and concepts associated with 'modern mathematics' content were discussed, together with suggestions for the development of the topics; examples of children's work were reproduced from time to time. *Computation and structure* addressed items such as natural numbers, counting numbers, place value, measures of length, weight, capacity, time and money, all of which could be regarded as traditional content. However, the major difference was the clear shift towards new ways of expressing and recording operations. For instance, mathematical mappings, estimation and inequalities were stressed as particularly important in operational terms, and the concept of sets, intersection and union and Venn diagrams, together with the related methodology of recording operations with sets, was strongly emphasised.

The literature of the Nuffield Primary Mathematics Project was endeavouring to re-educate teachers into a new kind of approach to mathematics, not only from the point of view of the content, but also in terms of advocating practical activity, group work and discussion to encourage pupils' understanding and concept formation. The significance of the work of the Project was immense in that it represented the first major cohesive thrust towards reforming mathematics education at the primary level in England and Wales for many decades.

J.K. Forgan produced a series of nine short workbooks of 34 pages each in 1963 for use in junior schools and entitled *Mathematics at work*. The content contained elements of both modern and traditional content but the approach was modern. A large number

of topics were introduced with a major recommendation that pupils should involve themselves in practical activity and be prepared to use pencil, ruler, scissors and paper to carry out the tasks which were linked to the various topics; these included work on graphs, coordinates, measurement, estimating areas by covering surfaces with non standard and standard measures and representing algebraic functions, such as the parabola, through curve stitching or drawing. In the advocacy of a change of content the books were perhaps not quite as advanced as the Nuffield Project books, but there were sufficient examples of new work and of a new approach to give them a place in this category.

### **Commentary and conclusions**

The principal conclusion drawn from these data is that schools, teachers and pupils were exposed to a powerful diet of traditional content material presented in a traditional manner for 30 items of the total of 51 in the sample. The 11 series at primary level in this category would be attractive to teachers in a junior school because they provided a ready-made course extending over at least two school years and possibly over four. At secondary level there were roughly an equal number of series and individual texts in this category, seven and eight respectively, and certainly, as at primary level, a series would be useful in that it addressed the total, or a high proportion of the requirement of the five year course across all the individual disciplines of mathematics such as algebra, trigonometry, geometry and arithmetic. The approximately equal mix of series and text in this category most probably reflected the particular wishes of the teachers in mathematics departments to deal with their subject matter according to their predilections, either through the utilisation of a general mathematics series or through the use of a number of discrete texts servicing particular aspects of the mathematics curriculum.

At the primary stage the substantial entry of 12 items in the traditional content/enhanced approach category would appear to reflect the advocacy of a change in the methodology of teaching generated by new thinking which was beginning to emerge at this time. Teachers were being encouraged to present material to pupils in a more 'user-friendly' style and to allow for an element of active participation. The books in this category, although still concentrating on traditional content with which authors, teachers and pupils were familiar, were colourful, attractive to handle and interactive in approach, and would lend considerable support to these methodological changes.

There is some evidence from the evidence that, at the secondary level, authors offering an enhanced content were prepared to adopt an enhanced approach in the presentation of their material. The conclusion must be drawn that these were the enthusiasts - able to convince teachers and students alike of the attraction of this new discipline. At the primary level only two

series were found in this category and one of those emanated from the Nuffield Primary Mathematics Project.

Research in the British Library to ascertain how many series of books or individual texts were reprinted indicated that this occurred in comparatively few cases - 13 instances in 51 in the total sample. However, where reprinting did occur the scope was often considerable. Flavell and Wakelam's book for primary schools *Mathematics: introduction to the language of number*, published from 1960 onwards, was a case in point. The series reflected a mix of principally traditional and a small amount of 'modern' content matched by a more user-friendly approach. Book 1 was reprinted five times from 1963 and the accompanying teachers' book three times.

It is significant that texts which could be seen as portraying traditional characteristics both in terms of content and approach were still being extensively reprinted at this time and hence were used to service the contemporary mathematics curriculum. A series entitled *Two-grade arithmetic* by Lovell and Smith, first published in 1956, ran to five reprints in subsequent years. Loney and Grenville's book, *Arithmetic*, was reprinted 31 times over 56 years, most

recently in 1962, whilst Hall and Stevens' *A school arithmetic* achieved 17 reprints over 53 years, with four between 1950 and 1961 - strong evidence of teachers' inclinations in respect of content and approach in mathematics education during this period.

In the enhanced content/traditional approach category Mansfield and Thompson's *Mathematics - a new approach* was reprinted substantially in the sense that Book 1 formed the basis of the first part of the teaching material for the highly successful Schools Mathematics Project.

The principal conclusion must be that if the biases revealed by this survey of books available in schools at this time reflect teachers' attitudes, a large number remained untouched by any attempts to modify either the mathematics content or the approach, or both. However, it is clear that a substantial number of primary schools were experimenting with changes in approach to teaching mathematics, whilst retaining a traditional curriculum. Some secondary schools were beginning to offer modern content in mathematics, suggesting that the main thrust for change in the body of knowledge to be taught was initially generated at this level.

#### List of books referred to:

- Anderson, K. 1961. *Arithmetic tests for 3rd year juniors* (London: Harrap)  
Bass, D. 1963. *Mathematics* (London: Cassell)  
Burn, E.W., Tamblin, D.V. 1962. *Arithmetic Itself* (Edinburgh: E.U.P.)  
Carter, V. A. 1961. *I work out* (London: Cassell)  
Ferrier, W. M. 1962. *Four Way Number* (London: E. J. Arnold)  
Flanagan, T.H. 1964. *Topics in Number* (London: Blackie)  
Flavell, J.S., Wakelam, B.B. 1960. *Primary Mathematics - An Introduction to the Language of number* (London: Methuen)  
Flavell, J.S., Wakelam, B.B. 1962- *Supplementary Books* (London: Methuen)  
Forgan, J.K. 1963. *Mathematics at Work* (Aylesbury: Ginn)  
Gibby, W.A. 1960. *General Certificate Tests* (Aylesbury: Ginn Odhams)  
Goddard, T.R., Grattidge, A.W. 1962. *Beta Junior Arithmetic* (Huddersfield: Schofield & Sims)  
Hall, H.S., Stevens, F.H. 1908. *A School Arithmetic* (London: Macmillan)  
Harris, R. E. 1961. *Cornerstone Mathematics* (London: Macmillan)  
Harris, R.E. 1953. *Practical Arithmetic for Girls* (London: Macmillan)  
Harris, R.E. 1964. *Angles* (London: Methuen)  
Hopkins, C.H. 1962. *Modern Arithmetic* (Harlow: Longmans)  
James, E. J. 1962. *Oxford Junior Mathematics* (Oxford: O. U. P.)  
James, W.S. 1961. *A Junior School Revision Course: Arithmetic and English* (London: Collins)  
Keith, A. 1962. *O Level Arithmetic* (London: Blackie)  
Larcombe, H.J. 1953. *A New Arithmetic for Primary School* (London: Macmillan)  
Lockwood, E.H. 1961. *A Book of Curves* (Cambridge: C.U.P.)  
Loney, S.L., Grenville, M.A. 1906. *Arithmetic* (London: Macmillan)  
Lovell, K., Smith, C.H.J. 1956. *Two-Grade Arithmetic* (Aylesbury: Odhams)  
Mansfield, D.E., Thompson, D. 1962. *Mathematics - A New Approach* (London: Chatto & Windus)  
Marsh, L.G. 1965. *Let's Explore Mathematics* (London: Black)

Matthews, G. (Ed) 1964. *Contemporary School Mathematics* (London: Edward Arnold) *Midland Mathematics Experiment*. 1963 (London: Harrap) *Nuffield Primary Mathematics Project Teacher's Guides* 1964 et seq (London: Chambers/Murray)

Razzell, A.G., Watts, K.G.O. 1964. *Mathematical Topics* (St. Albans: Hart Davis Educational)

Saunders, J.G. 1964. *Mathematics Alive* (London: Hamish Hamilton) Schools Mathematics Project, 1964 - SMP Books 1 – 5 (London: CUP)

Schools Mathematics Project, 1968 - SMP Books A - H (London: CUP)

Scottish Mathematics Group. 1956. *Modern Mathematics for Schools* (London: Blackie)

Sealey, L.G.W. 1961. *Some Important Mathematical Ideas* (Oxford: Basil Blackwell) Shaw, H.A., Wright, F.E. 1960. *Discovering Mathematics* (Walton on Thames: Edward Arnold)

Smith, T. 1955. *The Story of Measurement* (Oxford: Blackwell)

Taylor, T., Ingleby, T. 1963. *Measuring and Recording* (Harlow: Longmans)

Thompson, H.J. 1962 . *Colour Factor Mathematics* (Oxford: Heineman Educational)

Walker, R. 1960. *School Mathematics* (London: Harrap)

Ward, T.H. 1960. *Ordinary Level Algebra* (London: Harrap)

Whittaker, D.E. 1965. *Mathematics through Discovery* (London: Harrap)

Willbrook. 1960. *Discovery Mathematics for Top Infants and Lower Juniors* (Harlow: ESA)

An analysis of the data suggests that learners in the different countries are offered different mathematics and given different opportunities to learn that mathematics, both of which are influenced by textbook and teacher. In addition, it identifies pupil access to textbooks in England as a cause for concern. In England, Her Majesty's Inspectorate estimated that two-thirds of middle and secondary schools used a commercial mathematics scheme for Year 7 and Year 8 pupils (HMI, 1992) and Millett and Johnson (1996) argue that mathematics has long been regarded by many teachers in Britain as a subject for which the textbook, or commercial scheme, is the main resource. Nowadays education in Wales differs slightly from the system used in England. The statutory national key stage tests in Wales were, until 2000, the same as in England and were managed by the School Curriculum and Assessment Authority (SCAA). Instead, optional teacher assessment materials were provided to schools in 2003 for use in English, mathematics and Welsh. Primary and secondary education together are sometimes (in particular, in Canada and the United States) referred to as "K-12" education, (K is for kindergarten, 12 is for twelfth grade). Grammar schools in the United Kingdom. Primary schools in Great Britain. At five, the age at which education becomes compulsory, children in England and Wales go to infant schools until they are seven and then on to junior schools until the age of about eleven (also in Northern Ireland, twelve in Scotland). Primary schools in Scotland take all children between the ages of five and twelve. In Scotland primary pupils must be taught reading, writing, arithmetic, English, music, art and handwork, nature study, physical training and, at the appropriate stage, history, geography and written composition; girls must learn needlework. Religious instruction is also given.